Particle Injection into a Uniform Flow

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Nomenclature

a = particle diameter
C = characteristic length [see Eq. (7)]
C_D = particle drag coefficient (for a sphere herein)
Re = particle Reynold's number gased on rms relative velocity [see Eq. (5)]
s',s = sound speed and s'/V_{pi}', respectively
t',t = time and $t'V_{pi}'/C$, respectively
U',U = axial velocity and U'/V_{pi}', respectively
V',V = normal velocity and V'/V_{pi}', respectively
x',x = axial distance from injection point and x'/C, respectively
y',y = normal distance from injection and y'/C, respectively
μ = fluid viscosity
ρ = density

Subscripts

f = fluid
i = initial value
p = particle

Superscripts

$(\ )'$ = dimensional variable

Introduction

THE performance of advanced propulsion systems may in some instances be limited by the degree to which the fuel and airstreams mix. One means by which the fuel penetration in airbreathing combustors may be increased, is through the use of solid or liquid combustible additives. The purpose of the present analysis is to indicate the degree of penetration that may be achieved by this means.

To this end analytic and numerical solutions are presented for the perpendicular injection of particles into a uniform, constant-velocity flow. Three drag laws are considered: a) Stokes' drag, $C_DRe = 24$, yielding a closed-form solution; b) standard drag law for a sphere based on available experimental data[1]; and c) an empirical drag curve accounting for Mach number effects.[2] The latter two required numerical solutions and were programed for the IBM 360. The particles are assumed to be rigid spheres subject only to drag forces. The effect of the particles on the flow (non-negligible volume concentrations) was not taken into account. The boundary layer on the plate was also neglected.

The results obtained indicate that it may be possible to substantially increase over-all fuel penetration into airbreathing combustors by employing fuels containing solid (metallic) particles. Further study of this problem is, of course, needed and is currently in progress.

Method of Analysis

The equations of motion of the particle in cartesian coordinates are

$$\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 \rho_p \frac{dU_p'}{dt'} = \frac{C_DRe}{4}\pi\left(\frac{a}{2}\right)\mu\left(U_f' - U_p'\right) \quad (1)$$

$$\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 \rho_p \frac{dV_p'}{dt'} = \frac{C_DRe}{4}\pi\left(\frac{a}{2}\right)\mu\left(V_f' - V_p'\right) \quad (2)$$

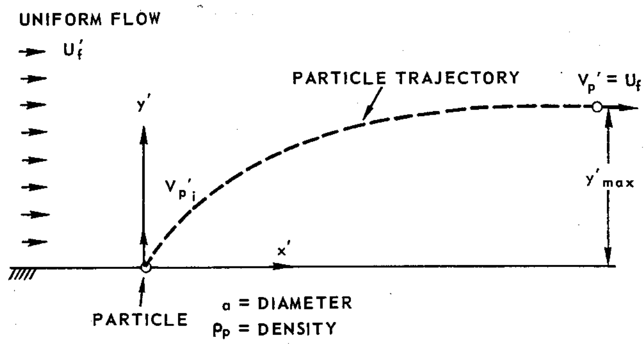

Fig. 1 Flow model sketch.

$$dx'/dt' = U_p' \quad (3)$$

$$dy'/dt' = V_p' \quad (4)$$

where

$$Re = \frac{\rho_f a}{\mu}\left[\left(U_f' - U_p'\right)^2 + \left(V_f' - V_p'\right)^2\right]^{1/2} \quad (5)$$

In nondimensional form the flow variables become

$$V_p = V_p'/V_{pi}',\ U_p = U_p'/V_{pi}',\ t = t'V_{pi}'/C, \quad (6)$$
$$V_f = V_f'/V_{pi}',\ U_f = U_f'/V_{pi}',\ x = x'/C,\ y = y'/C.$$

where V_{pi}' is the initial particle velocity and

$$C \equiv a^2\rho_p/18\mu V_{pi}' \quad (7)$$

Note that this parameter is the Stokes' penetration distance of a particle injected into a quiescent fluid with a given initial velocity, V_{pi}'. With this transformation, Eqs. (1–5) become

$$dU_p/dt = C_DRe(U_f - U_p)/24 \quad (8)$$

$$dV_p/dt = C_DRe(V_f - V_p)/24 \quad (9)$$

$$dx/dt = U_p,\ dy/dt = V_p \quad (10)$$

$$Re_p = Re_{pi}[(U_f - U_p)^2 + (V_f - V_p)^2]^{1/2} \quad (11)$$

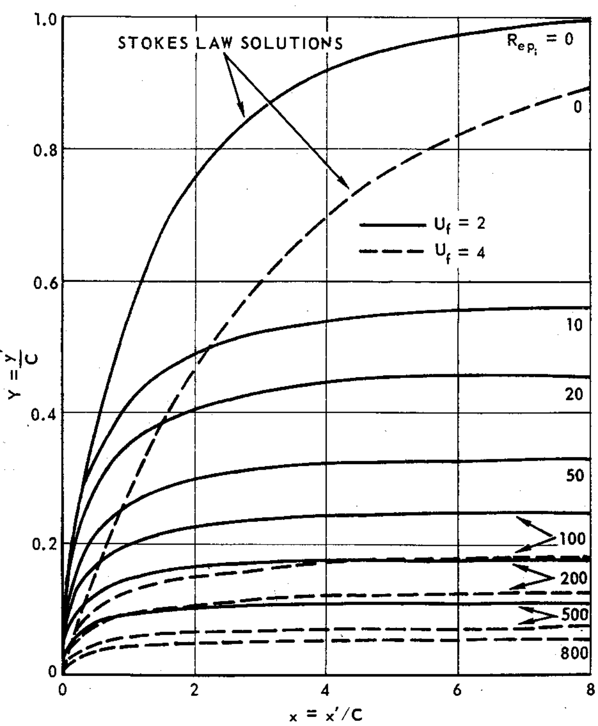

Fig. 2 Particle trajectories in normalized coordinates, standard drag.

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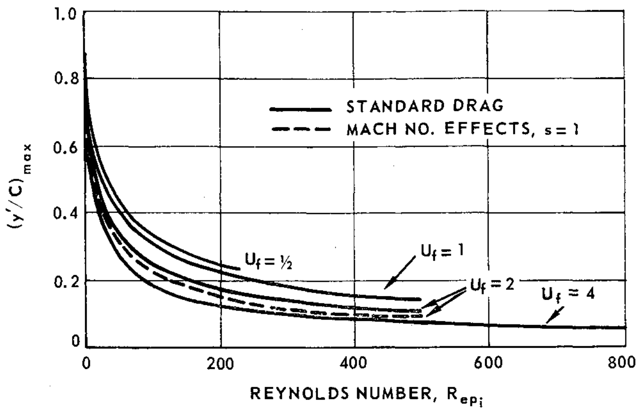

Fig. 3 Maximum penetration as a function of Reynolds number.

$$Re_{pi} = a\rho_f V_{pi}/\mu \qquad (12)$$

For the particular case (Fig. 1) of normal injection of a particle ($V_{pi} = 1$, $U_{pi} = 0$) into a uniform, axial stream (U_f = const, $V_f = 0$), if Stokes' drag law ($C_DRe = 24$) is assumed to hold, then Eqs. (8–10) can be integrated to yield

$$U_p = U_f(1 - e^{-t}),\ V_p = e^{-t} \qquad (13)$$

$$x = U_f t - (1 - e^{-t}),\ y = 1 - e^{-t} \qquad (14)$$

Thus, for the case in Fig. 1 and for the Stokes' law regime, the max penetration of the particle [from Eq. (14)] is

$$y_{max} = 1,\ y'_{max} = C \qquad (15)$$

Equations (8–12), with the conditions of Fig. 1, were numerically integrated for two other assumed drag law variations: the standard drag law,[1] and an empirical drag law accounting for compressibility and rarefaction (Mach number) effects.[2] For this latter case, the nondimensional velocities were divided by the nondimensional sound speed of the freestream, $s = s'/V_{pi}$ and for the purpose of illustration in the present study this value was chosen rather arbitrarily.

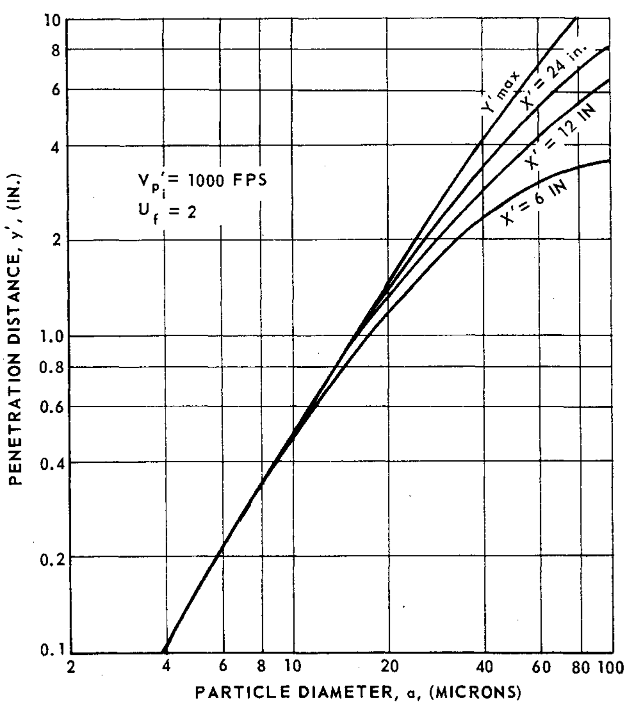

Fig. 4 Penetration distance as a function of particle diameter.

Results

Particle trajectories in normalized coordinates as a function of Re_{pi} are shown in Fig. 2 for $U_f = 2.0$ and 4.0. The $Re_{pi} = 0$ cases represent the Stokes' solution presented previously. Figure 3 summarizes the max values of y, y_{max} for the cases run.

As an illustration of the application of the results to a specific problem consider the injection of a 20μ diam particle with $V_{pi}' = 1000$ fps into a uniform air‡ flow at $U_f' = 2000$ fps ($U_f = 2$). It is desired to find the penetration at $z' = 1$ ft. From Eq. (12), $Re_{pi} = 110$, and from Eq. (7), $C = 0.5$ ft. Then from Fig. 2, $y = 0.215$, and $y' = y/C = 0.107$ ft. The maximum penetration for this case is found from Fig. 3 as $y_{max} = 0.235$, or $y'_{max} = 0.118$ ft $= 1.4$ in. Utilizing the aforementioned procedure, the results in Fig. 4 have been generated to show penetration depth as a function of axial location and particle diameter for the particular values of $U_f' = 2000$ fps and $V_{pi}' = 1000$ fps. Figure 4 illustrates that penetration depths at a distance of 24 in. from the injector would vary from, for example, 0.1 in. for a 4μ particle to 8 in. for a 100μ particle.

The results described previously were based on the standard drag curve. In order to assess the effect of compressibility the empirical relationship developed by Crowe[2] was utilized in a series of trajectories summarized in Fig. 3 by the curves for $s = 1.0$. As can be seen, the effect of compressibility is small but increases percentagewise as Re_{pi} increases and amounts to a 20% reduction at $Re_{pi} = 500$ for $U_f = 2$.

Discussion

The preceding analysis is directed at estimating penetration depths of solid particles with application to fuel injection in an airbreathing combustor (i.e., Ref. 3). In an actual combustor, the solid particles would probably be immersed in a gaseous stream. The particle trajectories would therefore be somewhat affected by the jet flowfield, including the attendant shock structure. These effects, however, would be minimal for larger particle sizes. Thus, aside from these complicating factors of the real problem the present results can be construed to indicate (see Figs. 3 and 4) that the mean fuel penetration may be substantially increased by utilizing particle laden fuel streams, where the particles are not too large to burn in a reasonable amount of time—say 10–20μ. Note that the mean penetration depth of a gaseous jet (without particles) is of the order of 0.3 in.[3] A practical scheme of solid fuel penetration (a heavily particle laden gaseous jet) could possibly employ a fuel rich metallized propellant solid rocket. Particle sizes would be controlled by propellant formulation techniques to give the desired particle size. The particle size desirable will ultimately result from a tradeoff of penetration distance (higher penetration with larger particles, as illustrated herein) and the particle combustion kinetics (larger particles require more time for combustion, see for example, Ref. 4). In addition, the use of metallized propellants would be advantageous because of their higher energy content (the usual reason for their use).

‡ The air is assumed to be $\rho_f = 0.0807$ lb/ft^3, $\mu = 1.5 \times 10^{-6}$ slugs/(fps) and the particle density was taken as $\rho_p = 100$ lb/ft^3.

References

[1] Schlichting, J., *Boundary Layer Theory*, 4th ed., McGraw-Hill, New York, 1960.

[2] Crowe, C. T., "Drag Coefficients of Particles in a Rocket Nozzle," *AIAA Journal*, Vol. 5, No. 5, May 1967, pp. 1021–1022.

[3] Orth, R. C. and Funk, J. A., "An Experimental and Comparative Study of Jet Penetration in Supersonic Flow," *Journal of Spacecraft and Rockets*, Vol. 4, No. 9, Sept. 1967, pp. 1256–1242.

[4] Prentice, J. L., ed., "Metal Particle Combustion Progress Report 1 July 1965-1 May 1967," NWC TP 4435 (AD 839 526), Aug. 1968, Naval Weapons Center, China Lake, Calif.